CONTRIBUTIONS TO KINEMATICS.

Classification of Bi-Circloids, Curves of two curvatures (in the same plane), *the Resultants of two circular movements.* By Henry Perigal, Jun., Esq., F.R.A.S.

" The *Doctrine of Motion* may be divided into two Sciences; Dynamics which treats of the *causes,* and Kinematics which treats of the *effects, of Motion.* The former developes *the movements which result from the forces* or causes treated of; the latter deduces *the laws of the modifications of Motion,* consequent upon the movements and combinations of movements treated of, *without reference to the forces* which produce and maintain the movements *." Here movement and motion are considered in the relation of cause and effect.

Kinematic Curves *are the resultants of combined movements;* the autoptical *representatives of the* Laws of Motion†. When delineated by instruments constructed for the purpose, they are the veritable *Autographs of Motion.* There are three systems by which they may thus be organically described : viz. *on a fixed plane by a moving point,* as Suardi's Geometric Pen ; *on a moving plane by a fixed point,* as Ibbetson's Geometric Chucks; and *on a moving plane by a moving point,* as Perigal's Kinegraphs, instruments contrived by myself. The same Curves, neither more nor less, can be produced by each of these methods : that is to say, no curve can be produced by either system that cannot likewise be produced by each of the others, though not by precisely the same arrangements of the combined movements.

Kinematic Curves may be classified according to *the number of circular movements* by which they are produced; each curve, of course, having *that* number of *curvatures.* Thus, the Circle is the only Curve of *one* curvature. The Curves of *two* curvatures, the resultants of two circular movements, are innumerable; so are the Curves of *three* curvatures, resulting from three circular movements; likewise the curves of *four* or more curvatures, which result from *four* or more circular movements ; each class comprising its own peculiar members, besides comprehending a proportion of those of the classes below. [The curvatures may be all in the *same plane,* or in two or more *different planes,* according to the circumstances of their generation : but at present I allude only to curvatures in *one* plane.] These may be termed Bi-Circloids,

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* *Notes on the Kinematic effects of* Revolution *and* Rotation. By H. Perigal, Jun., 1846–49.

† These laws are not to be confounded with "*the three Laws of Motion,*" attributed to Newton ; which sometimes appear to be regarded as if they were the *only* Laws of Motion !

TRI-CIRCLOIDS, TETRA-CIRCLOIDS, PENTA-CIRCLOIDS, &c., according to the number of generating movements.

Although the number of KINEMATIC CURVES in each class is known to be innumerable, yet it is interesting to estimate how many curves are comprised within certain limitations which may be assumed for the purpose: and a comparative notation of their integrant parts or symmetric branches offers facilities for such a ▮▮▮▮▮▮.

It is evident on inspection that *every Curve is compounded of a certain number of like parts,* which may be called the *limbs* of the Curve; their number being dependent upon the particular combination of movements that produced the curve. Thus the Oval (or Egg shape) may be divided into *two* symmetric halves, the Ellipse into *four* symmetric quarters, the Circle into *any number* of symmetric arcs, &c., &c.: such aggregational construction being still more apparent in the looped Epicycloids and Epitrochoids. It is on this principle that I have arranged the accompanying *Tables of* BICIRCLOIDS.

In the mathematical investigation of Curves, the first of the three systems above-mentioned seems to be considered the most convenient by mathematicians; who treat of Bicircloids as if generated by a point traversing the periphery of a circle called the *Epicycle,* while the center of the epicycle is carried round the circumference of another circle called the *Deferent,* with a constant ratio of velocity.

When the angular velocity of the Epicycle is *less* than that of the Deferent, the Curve progresses in *spires* (coils or circumvolutions) and may therefore be called *Spiroid, Spiralite,* or *Convolute.* When the angular velocity of the Epicycle is *greater* than that of the Deferent, and their radii are equal, the Bicircloid becomes a *looped* curve: the loops all meet in the center, and are *internal* or *external* according as the two movements are *direct* or *inverse*; that is to say, in the same or in contrary directions. When the ratio of the velocities is expressed by a fraction in its lowest terms, the *numerator* denotes the *number* of the Loops; and the *denominator* their *order,* whether *consecutive* or *alternate,* &c.

The Curves may be called *ciscentric, concentric* or *centric,* and *transcentric*; according as each of their component branches lies wholly *on one side* of the center, *cuts* the center, or *circumscribes* the center, of the Deferent.

The points most *distant* from, and *nearest* to, the center, are called *Apocenters* and *Pericenters,* the *Apses* of the curve; situated in two circles, the radii of which are equal respectively to the *sum* and to the *difference of the radii* of Epicycle and Deferent: the number of apocenters, as well as the number of pericenters, being the same as the number of branches.

Smith Street, Chelsea,   HENRY PERIGAL, Jun.
    August 1849.

BI-CIRCLOIDS.

Classification of *the first two hundred Curves resulting from two circular movements*:—each of which can be produced by *two different combinations**; excepting only the Bicircloid of *one external loop*, represented by (the *Annuloid* or *Dactyloid*) the *excentric Circle*, when the two movements are *equal* in angular velocity around their own centers and contrary in direction, the motion consequently *parallel*†; and the Bicircloid of *two external loops*, when the angular velocities are 2 : 1 in contrary directions, represented by the family of *Ellipses* including their extremes the *Circle* and (the *Rectoid* or *Orthoid*) the *finite Right-line*.

The Curves are arranged according to the number of their *Limbs* or symmetric *Branches* (integrant parts). With their polar Equations when *centric*; the radius of the Epicycle (the moving Circle) being equal to that of the Deferent (the fixed Circle):

$$r=a\cos\frac{V}{2\mathbf{V}+V}\theta,\ \text{direct; or}\quad r=a\cos\frac{V}{2\mathbf{V}\sim V}\theta,\ \text{inverse.}$$

No. of Curve.	Direct.	Inverse.	Polar Equation of centric curve.
1.	—	1:1,	$r=a\cos\theta$.†‡
2.	1:1	or 1:2,	$r=a\cos\frac{1}{3}\theta$.§
3.	—	2:1,	$r=a\cos\frac{2}{0}\theta$.‖
4.	2:1	or 2:3,	$r=a\cos\frac{1}{2}\theta$.
5.	1:2	or 1:3,	$r=a\cos\frac{1}{5}\theta$.
6. {	3:1	or 3:2,	$r=a\cos 3\theta$.
7.	3:1	or 3:4,	$r=a\cos\frac{3}{5}\theta$.
8.	1:3	or 1:4,	$r=a\cos\frac{1}{7}\theta$.
9.	3:2	or 3:5,	$r=a\cos\frac{3}{7}\theta$.
10.	2:3	or 2:5,	$r=a\cos\frac{1}{4}\theta$.
11. {	4:1	or 4:3,	$r=a\cos 2\theta$.
12.	4:1	or 4:5,	$r=a\cos\frac{2}{3}\theta$.
13.	1:4	or 1:5,	$r=a\cos\frac{1}{9}\theta$.
14.	4:3	or 4:7,	$r=a\cos\frac{2}{5}\theta$.
15.	3:4	or 3:7,	$r=a\cos\frac{3}{11}\theta$.
16. {	5:1	or 5:4,	$r=a\cos\frac{5}{3}\theta$.
17.	5:1	or 5:6,	$r=a\cos\frac{5}{7}\theta$.
18.	1:5	or 1:6,	$r=a\cos\frac{1}{11}\theta$.
19. {	5:2	or 5:3,	$r=a\cos 5\theta$.
20.	5:2	or 5:7,	$r=a\cos\frac{5}{9}\theta$.
21.	2:5	or 2:7,	$r=a\cos\frac{1}{6}\theta$.
22.	5:3	or 5:8,	$r=a\cos\frac{5}{11}\theta$.
23.	3:5	or 3:8,	$r=a\cos\frac{3}{13}\theta$.
24.	5:4	or 5:9,	$r=a\cos\frac{5}{13}\theta$.
25.	4:5	or 4:9,	$r=a\cos\frac{2}{7}\theta$.
26. {	6:1	or 6:5,	$r=a\cos\frac{3}{2}\theta$.
27.	6:1	or 6:7,	$r=a\cos\frac{3}{4}\theta$.

$a=(\mathbf{E}+\mathbf{D})=$ radius of Epicycle + radius of Deferent.

* $\frac{V}{\mathbf{V}}$ *inverse* or $\frac{V}{V-\mathbf{V}}$ *inverse*, complementary curves; and $\frac{V}{\mathbf{V}}$ *direct* or $\frac{V}{V+\mathbf{V}}$ *inverse*, supplemental curves: where $\frac{V}{\mathbf{V}}=\dfrac{\text{angular }\textit{velocity}\text{ of Epicycle}}{\text{angular }\textit{velocity}\text{ of Deferent}}$, each about its own center; *direct* when the two movements are in the same direction, *inverse* when contrary. ‡ The *Annuloid*. § The *Trisectris*. ‖ The *Orthoid*.

† The solitary case of *parallel* motion among Bicircloids; because the only possible instance of double circular motion which is *epi-reciprocal*.

No. of Curve.	Velocities Direct.	Velocities Inverse.	Polar Equation of centric Curve.
28.	1:6	1:7	$r=a\cos\frac{1}{13}\theta$
29.	6:5	6:11	$r=a\cos\frac{3}{8}\theta$
30.	5:6	5:11	$r=a\cos\frac{5}{17}\theta$
31.	7:1	7:6	$r=a\cos\frac{7}{5}\theta$
32.	7:1	7:8	$r=a\cos\frac{7}{9}\theta$
33.	1:7	1:8	$r=a\cos\frac{1}{15}\theta$
34.	7:2	7:5	$r=a\cos\frac{7}{3}\theta$
35.	7:2	7:9	$r=a\cos\frac{7}{11}\theta$
36.	2:7	2:9	$r=a\cos\frac{1}{8}\theta$
37.	7:3	7:4	$r=a\cos 7\theta$
38.	7:3	7:10	$r=a\cos\frac{7}{13}\theta$
39.	3:7	3:10	$r=a\cos\frac{3}{17}\theta$
40.	7:4	7:11	$r=a\cos\frac{7}{15}\theta$
41.	4:7	4:11	$r=a\cos\frac{2}{9}\theta$
42.	7:5	7:12	$r=a\cos\frac{7}{17}\theta$
43.	5:7	5:12	$r=a\cos\frac{5}{19}\theta$
44.	7:6	7:13	$r=a\cos\frac{7}{19}\theta$
45.	6:7	6:13	$r=a\cos\frac{3}{10}\theta$
46.	8:1	8:7	$r=a\cos\frac{4}{3}\theta$
47.	8:1	8:9	$r=a\cos\frac{4}{5}\theta$
48.	1:8	1:9	$r=a\cos\frac{1}{17}\theta$
49.	8:3	8:5	$r=a\cos 4\theta$
50.	8:3	8:11	$r=a\cos\frac{4}{7}\theta$
51.	3:8	3:11	$r=a\cos\frac{3}{19}\theta$
52.	8:5	8:13	$r=a\cos\frac{4}{9}\theta$
53.	5:8	5:13	$r=a\cos\frac{5}{21}\theta$
54.	8:7	8:15	$r=a\cos\frac{4}{11}\theta$
55.	7:8	7:15	$r=a\cos\frac{7}{23}\theta$
56.	9:1	9:8	$r=a\cos\frac{9}{7}\theta$
57.	9:1	9:10	$r=a\cos\frac{9}{11}\theta$
58.	1:9	1:10	$r=a\cos\frac{1}{19}\theta$
59.	9:2	9:7	$r=a\cos\frac{9}{5}\theta$
60.	9:2	9:11	$r=a\cos\frac{9}{13}\theta$
61.	2:9	2:11	$r=a\cos\frac{1}{10}\theta$
62.	9:4	9:5	$r=a\cos 9\theta$
63.	9:4	9:13	$r=a\cos\frac{9}{17}\theta$
64.	4:9	4:13	$r=a\cos\frac{2}{11}\theta$
65.	9:5	9:14	$r=a\cos\frac{9}{19}\theta$
66.	5:9	5:14	$r=a\cos\frac{5}{23}\theta$
67.	9:7	9:16	$r=a\cos\frac{9}{23}\theta$
68.	7:9	7:16	$r=a\cos\frac{7}{25}\theta$
69.	9:8	9:17	$r=a\cos\frac{9}{25}\theta$
70.	8:9	8:17	$r=a\cos\frac{4}{13}\theta$
71.	10:1	10:9	$r=a\cos\frac{5}{4}\theta$
72.	10:1	10:11	$r=a\cos\frac{5}{6}\theta$
73.	1:10	1:11	$r=a\cos\frac{1}{21}\theta$
74.	10:3	10:7	$r=a\cos\frac{5}{2}\theta$
75.	10:3	10:13	$r=a\cos\frac{5}{8}\theta$
76.	3:10	3:13	$r=a\cos\frac{3}{23}\theta$
77.	10:7	10:17	$r=a\cos\frac{5}{12}\theta$
78.	7:10	7:17	$r=a\cos\frac{7}{27}\theta$
79.	10:9	10:19	$r=a\cos\frac{5}{14}\theta$
80.	9:10	9:19	$r=a\cos\frac{9}{29}\theta$
81.	11:1	11:10	$r=a\cos\frac{11}{9}\theta$
82.	11:1	11:12	$r=a\cos\frac{11}{13}\theta$
83.	1:11	1:12	$r=a\cos\frac{1}{23}\theta$
84.	11:2	11:9	$r=a\cos\frac{11}{7}\theta$
85.	11:2	11:13	$r=a\cos\frac{11}{15}\theta$
86.	2:11	2:13	$r=a\cos\frac{1}{12}\theta$

$$a=(D+E)=2D=2E=\text{ radius of Apocentral Circle.}$$

No. of Curve	Velocities (Direct or Inverse)	Polar Equation of centric Curve
85.	11:2 or 11:13	$r=a\cos\frac{11}{15}\theta$
86.	2:11 or 2:13	$r=a\cos\frac{1}{12}\theta$
87.	{ 11:3 or 11:8	$r=a\cos\frac{11}{5}\theta$
88.	11:3 or 11:14	$r=a\cos\frac{11}{17}\theta$
89.	3:11 or 3:14	$r=a\cos\frac{3}{25}\theta$
90.	{ 11:4 or 11:7	$r=a\cos\frac{11}{3}\theta$
91.	11:4 or 11:15	$r=a\cos\frac{11}{19}\theta$
92.	4:11 or 4:15	$r=a\cos\frac{2}{13}\theta$
93.	{ 11:5 or 11:6	$r=a\cos 11\,\theta$
94.	11:5 or 11:16	$r=a\cos\frac{11}{21}\theta$
95.	5:11 or 5:16	$r=a\cos\frac{5}{27}\theta$
96.	11:6 or 11:17	$r=a\cos\frac{11}{23}\theta$
97.	6:11 or 6:17	$r=a\cos\frac{3}{14}\theta$
98.	11:7 or 11:18	$r=a\cos\frac{11}{25}\theta$
99.	7:11 or 7:18	$r=a\cos\frac{7}{29}\theta$
100.	11:8 or 11:19	$r=a\cos\frac{11}{27}\theta$
101.	8:11 or 8:19	$r=a\cos\frac{4}{15}\theta$
102.	11:9 or 11:20	$r=a\cos\frac{11}{29}\theta$
103.	9:11 or 9:20	$r=a\cos\frac{9}{31}\theta$
104.	11:10 or 11:21	$r=a\cos\frac{11}{31}\theta$
105.	10:11 or 10:21	$r=a\cos\frac{5}{16}\theta$
106.	{ 12:1 or 12:11	$r=a\cos\frac{6}{5}\theta$
107.	12:1 or 12:13	$r=a\cos\frac{6}{7}\theta$
108.	1:12 or 1:13	$r=a\cos\frac{1}{25}\theta$
109.	{ 12:5 or 12:7	$r=a\cos 6\,\theta$
110.	12:5 or 12:17	$r=a\cos\frac{6}{11}\theta$
111.	5:12 or 5:17	$r=a\cos\frac{5}{29}\theta$
112.	12:7 or 12:19	$r=a\cos\frac{6}{13}\theta$
113.	7:12 or 7:19	$r=a\cos\frac{7}{31}\theta$
114.	12:11 or 12:23	$r=a\cos\frac{6}{17}\theta$
115.	11:12 or 11:23	$r=a\cos\frac{11}{35}\theta$
116.	{ 13:1 or 13:12	$r=a\cos\frac{13}{11}\theta$
117.	13:1 or 13:14	$r=a\cos\frac{13}{15}\theta$
118.	1:13 or 1:14	$r=a\cos\frac{1}{27}\theta$
119.	{ 13:2 or 13:11	$r=a\cos\frac{13}{9}\theta$
120.	13:2 or 13:15	$r=a\cos\frac{13}{17}\theta$
121.	2:13 or 2:15	$r=a\cos\frac{1}{14}\theta$
122.	{ 13:3 or 13:10	$r=a\cos\frac{13}{7}\theta$
123.	13:3 or 13:16	$r=a\cos\frac{13}{19}\theta$
124.	3:13 or 3:16	$r=a\cos\frac{3}{29}\theta$
125.	{ 13:4 or 13:9	$r=a\cos\frac{13}{5}\theta$
126.	13:4 or 13:17	$r=a\cos\frac{13}{21}\theta$
127.	4:13 or 4:17	$r=a\cos\frac{2}{15}\theta$
128.	{ 13:5 or 13:8	$r=a\cos\frac{13}{3}\theta$
129.	13:5 or 13:18	$r=a\cos\frac{13}{23}\theta$
130.	5:13 or 5:18	$r=a\cos\frac{5}{31}\theta$
131.	{ 13:6 or 13:7	$r=a\cos 13\,\theta$
132.	13:6 or 13:19	$r=a\cos\frac{13}{25}\theta$
133.	6:13 or 6:19	$r=a\cos\frac{3}{16}\theta$
134.	13:7 or 13:20	$r=a\cos\frac{13}{27}\theta$
135.	7:13 or 7:20	$r=a\cos\frac{7}{33}\theta$
136.	13:8 or 13:21	$r=a\cos\frac{13}{29}\theta$
137.	8:13 or 8:21	$r=a\cos\frac{4}{17}\theta$
138.	13:9 or 13:22	$r=a\cos\frac{13}{31}\theta$
139.	9:13 or 9:22	$r=a\cos\frac{9}{35}\theta$
140.	13:10 or 13:23	$r=a\cos\frac{13}{33}\theta$
141.	10:13 or 10:23	$r=a\cos\frac{5}{18}\theta$
142.	13:11 or 13:24	$r=a\cos\frac{13}{35}\theta$
143.	11:13 or 11:24	$r=a\cos\frac{11}{37}\theta$
144.	13:12 or 13:25	$r=a\cos\frac{13}{37}\theta$
145.	12:13 or 12:25	$r=a\cos\frac{6}{19}\theta$

$a=(D+E)=$ radius of Circle which circumscribes the Curve.

No. of Curve.	Velocities (Direct : Inverse)	Polar Equation of centric Curve.
146.	{ 14:1 or 14:13,	$r=a\cos\frac{7}{6}\theta$.
147.	14:1 or 14:15,	$r=a\cos\frac{7}{8}\theta$.
148.	1:14 or 1:15,	$r=a\cos\frac{1}{29}\theta$.
149.	{ 14:3 or 14:11,	$r=a\cos\frac{7}{4}\theta$.
150.	14:3 or 14:17,	$r=a\cos\frac{7}{10}\theta$.
151.	3:14 or 3:17,	$r=a\cos\frac{3}{31}\theta$.
152.	{ 14:5 or 14:9,	$r=a\cos\frac{7}{2}\theta$.
153.	14:5 or 14:19,	$r=a\cos\frac{7}{12}\theta$.
154.	5:14 or 5:19,	$r=a\cos\frac{5}{33}\theta$.
155.	14:9 or 14:23,	$r=a\cos\frac{7}{16}\theta$.
156.	9:14 or 9:23,	$r=a\cos\frac{9}{37}\theta$.
157.	14:11 or 14:25,	$r=a\cos\frac{7}{18}\theta$.
158.	11:14 or 11:25,	$r=a\cos\frac{11}{39}\theta$.
159.	14:13 or 14:27,	$r=a\cos\frac{7}{20}\theta$.
160.	13:14 or 13:27,	$r=a\cos\frac{13}{41}\theta$.
161.	{ 15:1 or 15:14,	$r=a\cos\frac{15}{13}\theta$.
162.	15:1 or 15:16,	$r=a\cos\frac{15}{17}\theta$.
163.	1:15 or 1:16,	$r=a\cos\frac{1}{31}\theta$.
164.	{ 15:2 or 15:13,	$r=a\cos\frac{15}{11}\theta$.
165.	15:2 or 15:17,	$r=a\cos\frac{15}{19}\theta$.
166.	2:15 or 2:17,	$r=a\cos\frac{1}{16}\theta$.
167.	{ 15:4 or 15:11,	$r=a\cos\frac{15}{7}\theta$.
168.	15:4 or 15:19,	$r=a\cos\frac{15}{23}\theta$.
169.	4:15 or 4:19,	$r=a\cos\frac{2}{17}\theta$.
170.	{ 15:7 or 15:8,	$r=a\cos 15\theta$.
171.	15:7 or 15:22,	$r=a\cos\frac{15}{29}\theta$.
172.	7:15 or 7:22,	$r=a\cos\frac{7}{37}\theta$.
173.	15:8 or 15:23,	$r=a\cos\frac{15}{31}\theta$.
174.	8:15 or 8:23,	$r=a\cos\frac{4}{19}\theta$.
175.	15:11 or 15:26,	$r=a\cos\frac{15}{37}\theta$.
176.	11:15 or 11:26,	$r=a\cos\frac{11}{41}\theta$.
177.	15:13 or 15:28,	$r=a\cos\frac{15}{41}\theta$.
178.	13:15 or 13:28,	$r=a\cos\frac{13}{43}\theta$.
179.	15:14 or 15:29,	$r=a\cos\frac{15}{43}\theta$.
180.	14:15 or 14:29,	$r=a\cos\frac{7}{22}\theta$.
181.	{ 16:1 or 16:15,	$r=a\cos\frac{8}{7}\theta$.
182.	16:1 or 16:17,	$r=a\cos\frac{8}{9}\theta$.
183.	1:16 or 1:17,	$r=a\cos\frac{1}{33}\theta$.
184.	{ 16:3 or 16:13,	$r=a\cos\frac{8}{5}\theta$.
185.	16:3 or 16:19,	$r=a\cos\frac{8}{11}\theta$.
186.	3:16 or 3:19,	$r=a\cos\frac{3}{35}\theta$.
187.	{ 16:5 or 16:11,	$r=a\cos\frac{8}{3}\theta$.
188.	16:5 or 16:21,	$r=a\cos\frac{8}{13}\theta$.
189.	5:16 or 5:21,	$r=a\cos\frac{5}{37}\theta$.
190.	{ 16:7 or 16:9,	$r=a\cos 8\theta$.
191.	16:7 or 16:23,	$r=a\cos\frac{8}{15}\theta$.
192.	7:16 or 7:23,	$r=a\cos\frac{7}{39}\theta$.
193.	16:9 or 16:25,	$r=a\cos\frac{8}{17}\theta$.
194.	9:16 or 9:25,	$r=a\cos\frac{9}{41}\theta$.
195.	16:11 or 16:27,	$r=a\cos\frac{8}{19}\theta$.
196.	11:16 or 11:27,	$r=a\cos\frac{11}{43}\theta$.
197.	16:13 or 16:29,	$r=a\cos\frac{8}{21}\theta$.
198.	13:16 or 13:29,	$r=a\cos\frac{13}{45}\theta$.
199.	16:15 or 16:31,	$r=a\cos\frac{8}{23}\theta$.
200.	15:16 or 15:31,	$r=a\cos\frac{15}{47}\theta$.

$a = (\mathbf{D} + \mathbf{E}) =$ radius of Apocentral Circle.

Every one of the above is susceptible of innumerable variations of form (phases), *dependent upon the ratio of the radius* of the Epicycle to that of the Deferent, when not *centric* or *equiradial.*

Polar Equation of centric Curve.	Ratio of Velocities. Inverse. / Inverse.	Polar Equation of centric Curve.	Ratio of Velocities. Direct. / Inverse.
$r = a \cos \theta$,	— 1:1.	$r = a \cos \theta$,	— 1:1.
$r = a \cos 2\theta$,	4:1 or 4:3.	$r = a \cos \frac{1}{2}\theta$,	2:1 or 2:3.
$r = a \cos 3\theta$,	3:1 or 3:2.	$r = a \cos \frac{1}{3}\theta$,	1:1 or 1:2.
$r = a \cos \frac{3}{2}\theta$,	6:1 or 6:5.	$r = a \cos \frac{2}{3}\theta$,	4:1 or 4:5.
$r = a \cos 4\theta$,	8:3 or 8:5.	$r = a \cos \frac{1}{4}\theta$,	2:3 or 2:5.
$r = a \cos \frac{4}{3}\theta$,	8:1 or 8:7.	$r = a \cos \frac{3}{4}\theta$,	6:1 or 6:7.
$r = a \cos 5\theta$,	5:2 or 5:3.	$r = a \cos \frac{1}{5}\theta$,	1:2 or 1:3.
$r = a \cos \frac{5}{2}\theta$,	10:3 or 10:7.	$r = a \cos \frac{2}{5}\theta$,	4:3 or 4:7.
$r = a \cos \frac{5}{3}\theta$,	5:1 or 5:4.	$r = a \cos \frac{3}{5}\theta$,	3:1 or 3:4.
$r = a \cos \frac{5}{4}\theta$,	10:1 or 10:9.	$r = a \cos \frac{4}{5}\theta$,	8:1 or 8:9.
$r = a \cos 6\theta$,	12:5 or 12:7.	$r = a \cos \frac{1}{6}\theta$,	2:5 or 2:7.
$r = a \cos \frac{6}{5}\theta$,	12:1 or 12:11.	$r = a \cos \frac{5}{6}\theta$,	10:1 or 10:11.
$r = a \cos 7\theta$,	7:3 or 7:4.	$r = a \cos \frac{1}{7}\theta$,	1:3 or 1:4.
$r = a \cos \frac{7}{2}\theta$,	14:5 or 14:9.	$r = a \cos \frac{2}{7}\theta$,	4:5 or 4:9.
$r = a \cos \frac{7}{3}\theta$,	7:2 or 7:5.	$r = a \cos \frac{3}{7}\theta$,	3:2 or 3:5.
$r = a \cos \frac{7}{4}\theta$,	14:3 or 14:11.	$r = a \cos \frac{4}{7}\theta$,	8:3 or 8:11.
$r = a \cos \frac{7}{5}\theta$,	7:1 or 7:6.	$r = a \cos \frac{5}{7}\theta$,	5:1 or 5:6.
$r = a \cos \frac{7}{6}\theta$,	14:1 or 14:13.	$r = a \cos \frac{6}{7}\theta$,	12:1 or 12:13.
$r = a \cos 8\theta$,	16:7 or 16:9.	$r = a \cos \frac{1}{8}\theta$,	2:7 or 2:9.
$r = a \cos \frac{8}{3}\theta$,	16:5 or 16:11.	$r = a \cos \frac{3}{8}\theta$,	6:5 or 6:11.
$r = a \cos \frac{8}{5}\theta$,	16:3 or 16:13.	$r = a \cos \frac{5}{8}\theta$,	10:3 or 10:13.
$r = a \cos \frac{8}{7}\theta$,	16:1 or 16:15.	$r = a \cos \frac{7}{8}\theta$,	14:1 or 14:15.
$r = a \cos 9\theta$,	9:4 or 9:5.	$r = a \cos \frac{1}{9}\theta$,	1:4 or 1:5.
$r = a \cos \frac{9}{2}\theta$,	18:7 or 18:11.	$r = a \cos \frac{2}{9}\theta$,	4:7 or 4:11.
$r = a \cos \frac{9}{4}\theta$,	18:5 or 18:13.	$r = a \cos \frac{4}{9}\theta$,	8:5 or 8:13.
$r = a \cos \frac{9}{5}\theta$,	9:2 or 9:7.	$r = a \cos \frac{5}{9}\theta$,	5:2 or 5:7.
$r = a \cos \frac{9}{7}\theta$,	9:1 or 9:8.	$r = a \cos \frac{7}{9}\theta$,	7:1 or 7:8.
$r = a \cos \frac{9}{8}\theta$,	18:1 or 18:17.	$r = a \cos \frac{8}{9}\theta$,	16:1 or 16:17.
$r = a \cos 10\theta$,	20:9 or 20:11.	$r = a \cos \frac{1}{10}\theta$,	2:9 or 2:11.
$r = a \cos \frac{10}{3}\theta$,	20:7 or 20:13.	$r = a \cos \frac{3}{10}\theta$,	6:7 or 6:13.
$r = a \cos \frac{10}{7}\theta$,	20:3 or 20:17.	$r = a \cos \frac{7}{10}\theta$,	14:3 or 14:17.
$r = a \cos \frac{10}{9}\theta$,	20:1 or 20:19.	$r = a \cos \frac{9}{10}\theta$,	18:1 or 18:19.
$r = a \cos 11\theta$,	11:5 or 11:6.	$r = a \cos \frac{1}{11}\theta$,	1:5 or 1:6.
$r = a \cos \frac{11}{2}\theta$,	22:9 or 22:13.	$r = a \cos \frac{2}{11}\theta$,	4:9 or 4:13.
$r = a \cos \frac{11}{3}\theta$,	11:4 or 11:7.	$r = a \cos \frac{3}{11}\theta$,	3:4 or 3:7.
$r = a \cos \frac{11}{4}\theta$,	22:7 or 22:15.	$r = a \cos \frac{4}{11}\theta$,	8:7 or 8:15.

$$\frac{r}{a} = \cos n\theta = \cos \frac{V}{2\mathbf{V} + V}\,\theta,\ \text{direct}; \quad \text{or}\quad \frac{r}{a} = \cos \frac{V}{2\mathbf{V} - V}\,\theta,\ \text{inverse}.$$

As all Bicircloids when *concentric* are looped curves, they may be classified according to the number of their *Loops* or of their *Spires*, their Limbs or symmetric Branches (integrant parts), determined by the velocity ratio.   In this way we may ascertain how many Bicircloids are comprised within certain limitations of loops and spires; as in the following Table. [Bicircloids are otherwise *innumerable*, because the ratios on which they depend are naturally inexhaustible.]

Let N be the given number; P the *primes* to it less than itself; R the *ratios* they will represent fractionally, and c the bicircloid *curves* whose velocities can be expressed thereby; *a, b, c, d,* &c., the *aliquot* parts of N: then

$$2N \cdot \left(\frac{a-1}{a} \cdot \frac{b-1}{b} \cdot \frac{c-1}{c} \cdot \frac{d-1}{d} \cdots \right) = R; \text{ and } R + \frac{R}{4} = C.$$

BI-CIRCLOIDS,

of which the Loops or Spires are

only	1,	=	2.	not more than	3,	=	10.
...	2,	=	3.	...	5,	=	25.
...	3,	=	5.	...	7,	=	45.
...	4,	=	5.	...	9,	=	70.
...	5,	=	10.	...	19,	=	300.
...	6,	=	5.	...	29,	=	675.
...	7,	=	15.	...	39,	=	1 185.
...	8,	=	10.	...	49,	=	1 885.
...	9,	=	15.	...	59,	=	2 715.
...	10,	=	10.	...	69,	=	3 675.
...	11,	=	25.	...	79,	=	4 835.
...	12,	=	10.	...	89,	=	6 140.
...	13,	=	30.	...	99,	=	7 510.
...	14,	=	15.	...	199,	=	30 380.
...	15,	=	20.	...	299,	=	68 295.
...	16,	=	20.	...	399,	=	121 295.
...	17,	=	40.	...	499,	=	189 790.
...	18,	=	15.	...	599,	=	273 350.
...	19,	=	45.	...	699,	=	371 945.
...	20,	=	20.	...	799,	=	486 075.
...	21,	=	30.	...	899,	=	615 215.
...	22,	=	25.	...	999,	=	759 480.
...	23,	=	55.	...	1 999,	=	3 038 540.
...	24,	=	20.	...	2 999,	=	6 837 540.
...	25,	=	50.	...	3 999,	=	12 154 435.
				...	4 999,	=	18 993 410.

Total under 26 = 500. | not more than 5 000, = 18 998 410.

Each of the above is susceptible of innumerable *phases*, or *variations of form*; dependent upon the adjustment of the variable element, the radial ratio.

Smith Street, Chelsea,   HENRY PERIGAL, Jun.
          August 1849.